Trade-offs between More and Less Concrete Manipulatives

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Abstract

How does access to concrete manipulatives affect children's performance on mathematical word problems? In Experiment 1, fourth- and sixth-grade children ($N = 229$) solved mathematics word problems involving money. In the *manipulatives* condition, children were given manipulatives. In the *no manipulatives* condition, children were not given manipulatives. Children in the manipulatives condition solved fewer problems correctly. Experiment 2 tested whether this effect was due to the highly concrete nature of the manipulatives. Fifth-grade children ($N = 79$) were given: (a) highly concrete manipulatives, (b) less concrete manipulatives, or (c) no manipulatives. Children in the highly concrete condition made more errors than did children in the other two conditions; however, their errors were less likely to be *conceptual* errors. Results suggest that although highly concrete manipulatives have the potential to support conceptual understanding, they also may increase the total number of errors.

Introduction

Question
Do children perform better on math problems when they have access to concrete manipulatives?

Alternative A
Yes, concrete manipulatives activate children's real-world knowledge and provide support for informal reasoning strategies.

A number of studies have shown that children perform better when they are able to draw on their of practical, real-world knowledge (e.g., Baranes et al., 1989; Carraher et al., 1985; Sternberg et al., 1998).

Alternative B
No, concrete manipulatives draw children's attention to the objects themselves instead of the mathematics at hand.

Several lines of research and theory suggest that highly concrete and perceptually rich stimuli can hinder learning and performance (Goldstone & Sakamoto, 2003; Sloutsky et al., 2005; Uttal et al., 1997).

Task
Mathematics word problems

It has been suggested that children perform poorly on math word problems because most academic testing situations do not encourage students to invoke their real-world knowledge (Carpenter et al., 1980; Cummins et al., 1988; Schoenfeld, 1989).

Example word problems

Charles buys a gumball for $0.20 and a candy bar for $1.15. If he gives the cashier $5.00, how much change does he get back?

Patricia and James are going shopping for their mom at the bakery. They were told to buy 18 onion rolls, 12 hamburger buns, and 2 loaves of rye bread. How much will it cost them if the following prices apply?

- White bread loaves – 2 for $1.25
- Rye bread loaves – 2 for $1.35
- Onion rolls – 6 for $1.00
- Hamburger buns – 6 for $0.85
- Hot dog buns – 6 for $0.69

Experiment 1

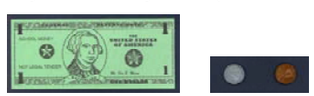
Method

Participants – 113 4th & 116 6th graders (111 boys & 118 girls)

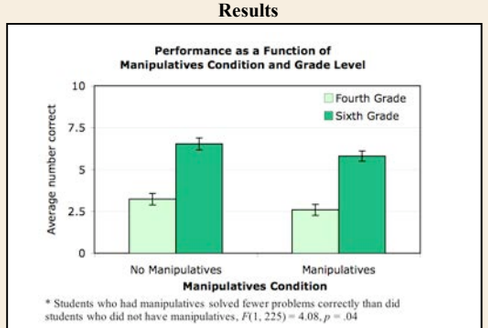
Procedure – 10 word problems involving money were administered as paper-and-pencil test in group settings

Conditions
Manipulatives – Bills and coins were provided
No manipulatives – Bills and coins were NOT provided

Examples of the manipulatives (bills and coins)



Results



Experiment 2

Method

Participants – 85 5th graders (47 boys & 38 girls)

Procedure – Same as Experiment 1, except children were encouraged to show their work on every problem, and they were shown an example of how to do so

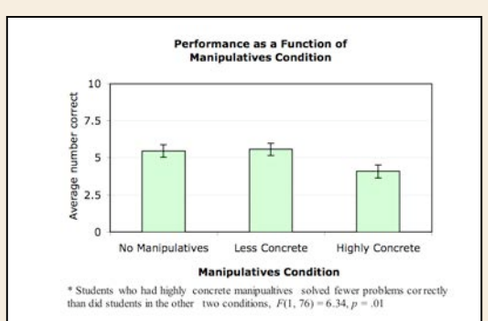
Conditions
No manipulatives – Bills and coins were NOT provided
Highly concrete manipulatives – Bills and coins from Exp 1
Less concrete manipulatives – Black-on-white bills and coins

Examples of the less concrete manipulatives

Coding
Correctness – Same DV as Exp 1
Conceptual errors – Errors that demonstrate a conceptual misunderstanding about the problem. For example, the most common conceptual error on the Charles problem (see word problems above left) was to add up all the numbers in the problem (i.e., $0.20 + $1.15 + $5.00 = $6.35). Contrast this with non-conceptual errors such as arithmetic errors, copy slips, etc.

Results

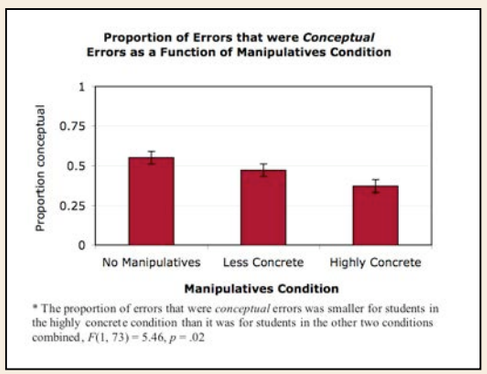
Correctness



Experiment 2 (cont.)

Results (cont.)

Conceptual errors



Summary and Conclusions

Concrete manipulatives increase overall number of errors
Concrete manipulatives require dual representation, which may lead children to focus on the manipulatives as objects themselves instead of on the math at hand (Uttal et al., 1997).

Dual representation is difficult for a number of reasons including: (a) the non-transparent mappings between manipulatives and the math ideas they symbolize, (b) children's limited cognitive resources, and (c) children's tendency to resist change.

Concrete manipulatives decrease prop of conceptual errors
Concrete manipulatives may activate real-world knowledge, which can help children capitalize on their informal reasoning skills and generate sensible solutions (Carraher et al., 1987).

Bottom line
There are both costs and benefits to using highly concrete manipulatives. These costs and benefits should be considered carefully when making decisions about the use of manipulatives in the classroom.

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