Arithmetic Practice that Promotes Conceptual Understanding and Computational Fluency

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Abstract

Most elementary school children struggle to understand mathematical equivalence, a concept necessary for success in algebra. This experiment tests whether or not children benefit from practice with arithmetic problems presented in nontraditional problem formats (e.g., __ = 3 + 4). Children (*M* age = 8;2) receive practice with arithmetic and complete tests to assess their understanding of mathematical equivalence. Children are randomly assigned to one of three practice conditions: (a) traditional, in which problems are presented in the traditional "operations = answer" format, such as 3 + 4 = __, (b) nontraditional, in which problems are presented in a nontraditional format, such as __ = 3 + 4, or (c) no-input control. Preliminary results indicate that children in the nontraditional condition exhibit a significantly better understanding of mathematical equivalence than children

Background

Mathematical equivalence is a fundamental concept in algebra, and success in algebra is crucial to future educational and employment opportunities.

Unfortunately, most children (ages 7-11) do not have a good understanding of mathematical equivalence. Misconceptions are robust and long term, persisting among high school and even college students.

We argue that difficulties with math equivalence are due to children's overly narrow experience with arithmetic in elementary school. Arithmetic is taught in a procedural fashion, with little or no reference to the equal sign or math equivalence. Problems are typically presented in a vertical format (e.g., $\begin{array}{r} 3 \\ +\,4 \\ \hline \end{array}$) or in a left-to-right format (e.g., 3 + 4 = __), neither of which highlights the interchangeable nature of the two sides of an equation.

Our research has shown that children pick up on 3 patterns from their experience. First, they learn that the equal sign and answer always come together at the end of a problem. Second, they learn to interpret the equal sign as an operator (like + or –) that means "calculate the total". Third, they learn to solve math problems by performing all given operations on all given numbers.

Background (cont.)

According to our account, children may benefit from practicing arithmetic in ways that conflict with these narrow patterns. We will be performing several experiments over the next few years to test this idea. In our first experiment, we test the effect of modifying the problem format. We hypothesize that practice with nontraditional problem formats (e.g., __ = 3 + 4) will lead to a better understanding of mathematical equivalence than will practice with the traditional problem format (e.g., 3 + 4 = __).

Method

Participants to date

53 children (*M* age = 8 yrs, 2 months; 24 girls, 29 boys; 30% Black, 11% Hispanic, 58% White)

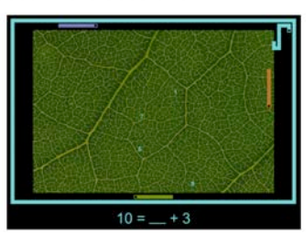
Procedure

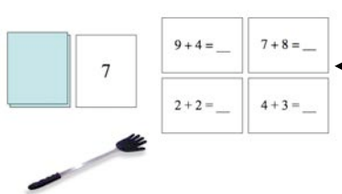
Children play math games and answer flashcards that use either a nontraditional or traditional problem format during three 30-minute one-on-one sessions with a tutor. In between sessions, children are asked to complete short paper-and-pencil assignments.

Children are assessed on their understanding of math equivalence during the third session. Children later complete a five-minute follow-up assessment approximately two weeks after the third session.

Example of Games Used in the Sessions

Snakey Math Computer Game curryk.com (nontrad version)





Smack it! Card Game (trad version)

*We have traditional & nontraditional versions of all games.

Method (cont.)

Conditions

Nontraditional– Problems presented in a nontraditional format (e.g., __ = 3 + 4, 7 = 3 + __). Note: none have operations on both sides of the equal sign.

Traditional– Problems presented in the traditional "operations = answer" format (e.g., 3 + 4 = __).

No-input control– Children complete the assessments (described in next section) before receiving practice.

Assessments

Understanding of mathematical equivalence:

- Equation-solving performance– Solve and explain math equivalence problems (e.g., 1 + 5 = __ + 2, 7 + 2 + 4 = __ + 4)
- Equation encoding– Reconstruct math equivalence problems after viewing for 5 sec.
- Equal sign understanding– Define the equal sign and rate fictitious students' definitions as "very smart," "kind of smart," or "not so smart"

Computational fluency:

- Math Computation section of ITBS Level 8
- Single-digit addition facts (RT and strategy)

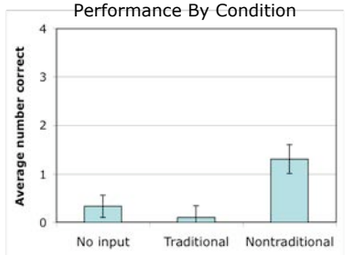
Follow up:

- Solve mathematical equivalence problems (with brief tutelage and feedback)

Results

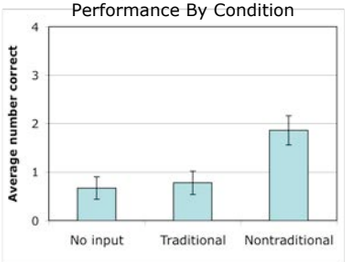
Results thus far show that children who practice problems with nontraditional formats perform better across all measures of understanding of mathematical equivalence than children in the other two conditions (see graphs).

Equation-Solving Performance By Condition

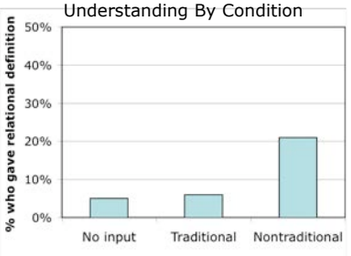


Results (cont.)

Equation Encoding Performance By Condition



Equal Sign Understanding By Condition



Follow-up

After tutelage and feedback, 75% of children who had received practice with nontraditional formats solved at least one equation correctly (compared to only 37% who had received practice with the traditional format).

Summary and Conclusions

Consistent with our hypothesis, preliminary results indicate that children construct a better understanding of mathematical equivalence after practicing arithmetic problems presented in a nontraditional format (versus the traditional format).

These findings support the view that difficulties with mathematical equivalence are due to children's overly narrow experience with arithmetic in elementary school.

It may be beneficial for teachers to introduce the nontraditional format into their classrooms as a way of improving students' understanding of mathematical equivalence and thus increasing algebra readiness.

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