Arithmetic Practice that Promotes Conceptual Understanding and Computational Fluency: Year 2

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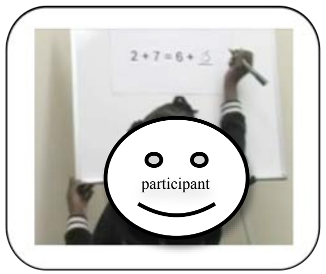
Abstract

We are performing several experiments to test if arithmetic practice can be optimized to improve understanding of math equivalence, a concept necessary for success in algebra. In the present experiment, we tested if children benefit from practice with arithmetic problems grouped by equivalent values. Children (*M* age = 8;6) received practice with arithmetic and completed tests to assess their understanding of math equivalence. Children were randomly assigned to one of three practice conditions: (a) nontraditional, in which problems were grouped by equivalent values (e.g., sums equal to 10 [1 + 9 = __, 2 + 8 = __, etc.]), (b) traditional, in which problems were grouped iteratively according to the counting sequence (e.g., all the ones [1 + 1 = __, 1 + 2 = __, etc.]), or (c) no-input control. Results indicate that children in the nontraditional condition exhibit a significantly better understanding of math equivalence than children in the other two conditions.

Background

Mathematical equivalence is a fundamental concept in algebra, and success in algebra is crucial to future educational and employment opportunities.



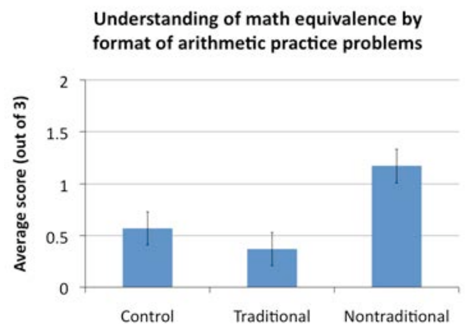
Unfortunately, most children (ages 7-11) do not have a good understanding of mathematical equivalence. Misconceptions are robust and long term, persisting among high school and even college students.

We argue that difficulties with math equivalence are due to children's overly narrow experience with arithmetic in elementary school. Arithmetic is taught in a procedural fashion, with little or no reference to the equal sign or math equivalence. Problems are almost always presented with the operations on the left side and the "answer" on the right (e.g., 3 + 4 = __), and problems are typically introduced iteratively according to a traditional addition table (all the ones, followed by all the twos, and so on), which may not help children induce equivalent added pairs (e.g., if 3 + 4 = 7 and 5 + 2 = 7, then 3 + 4 = 5 + 2).

According to our account, arithmetic practice that is modified to be less narrow will promote understanding of math equivalence. We are performing several experiments to test this idea.

Background (cont.)

In our first experiment (presented last year), we tested the effect of modifying the problem format. We hypothesized that practice with nontraditional problem formats (e.g., __ = 3 + 4) would promote a better understanding of math equivalence than would practice with the traditional format (e.g., 3 + 4 = __). Results supported this hypothesis. Children constructed a better understanding of math equivalence after practicing problems presented in a nontraditional (vs. traditional) format (see graph below).



In the experiment we are presenting this year, we tested the effect of modifying how problems are grouped during practice. We hypothesized that a nontraditional grouping in which problems are grouped by equivalent values would lead to a better understanding of math equivalence than would practice with problems grouped iteratively according to a traditional additional table.

Method

Participants to date

104 children (*M* age = 8;6; 55 girls, 49 boys; 2% Asian, 26% African American or Black, 12% Hispanic or Latino, 57% White, 3% Other).

Procedure

Children play math games and answer flashcards that use either a nontraditional or traditional way of grouping the problems during three 30-minute one-on-one sessions with a tutor. In between sessions, children are asked to complete short paper-and-pencil assignments.

Children are assessed on their understanding of math equivalence during the third session. Children later complete a five-minute follow-up assessment approximately two weeks after the third session.

Method (cont.)

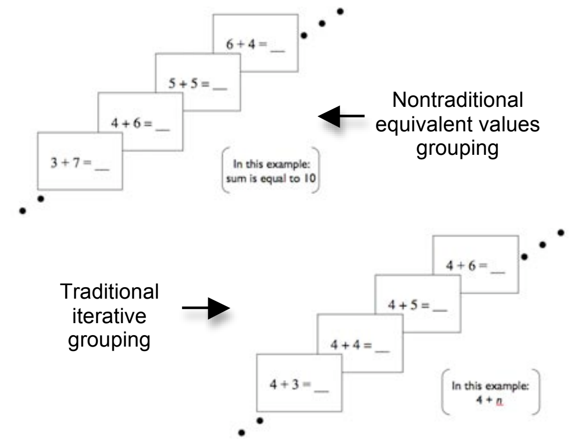
Conditions

Nontraditional grouping– Problems grouped by equivalent values (e.g., sums equal to 10 [1 + 9 = __, 2 + 8 = __, 3 + 7 = __, etc.]).

Traditional grouping– Problems grouped iteratively according to the counting sequence (e.g., all the ones [1 + 1 = __, 1 + 2 = __, etc.], all the twos [2 + 1 = __, 2 + 2 = __, etc.], etc.).

No-input control– Children complete the assessments (described in next section) before receiving practice.

Example of Nontraditional and Traditional Problem Grouping During Flashcard Practice



*We have nontrad. & trad. versions of all activities.

Assessments

Understanding of mathematical equivalence:

- Equation-solving performance– Solve and explain math equivalence problems (e.g., 1 + 5 = __ + 2)
- Equation encoding– Reconstruct math equivalence problems after viewing for 5 sec.
- Equal sign understanding– Define the equal sign

Computational fluency:

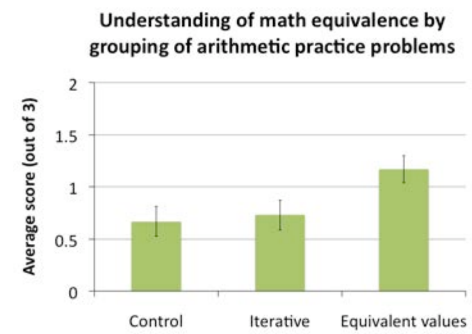
- Math Computation section of ITBS Level 8
- Single-digit addition facts (RT and strategy)

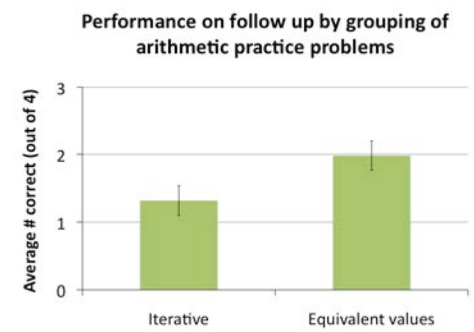
Follow up:

- Solve mathematical equivalence problems (with brief tutelage and feedback)

Results

Results show that children who practice problems grouped nontraditionally construct a better understanding of mathematical equivalence than children in the other two conditions (see graphs below).





Summary and Conclusions

Consistent with our hypothesis, results indicate that children construct a better understanding of mathematical equivalence after practicing arithmetic problems grouped by equivalent values (versus iteratively).

These findings support the view that children can benefit from relatively minor modifications that broaden their arithmetic practice.

It may be beneficial for teachers to introduce problem sets that are grouped by equivalent values into their classrooms as a way of improving children's understanding of math equivalence and thus increasing algebra readiness.

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