Benefits of "Concreteness Fading" for Children with Low Knowledge of Mathematical Equivalence

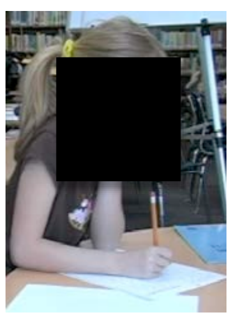
Emily R. Fyfe and Nicole M. McNeil, University of Notre Dame

Abstract

A longstanding controversy concerns the use of generalized, abstract symbols versus perceptually rich, concrete materials to facilitate learning. Although recent evidence suggests that concrete materials may hinder transfer relative to abstract symbols, many theorists specifically recommend beginning with concrete materials and fading to the more abstract. We tested this "fading hypothesis" in the context of children learning mathematical equivalence. Children who had low knowledge of mathematical equivalence were given instruction in one of four conditions: (a) concrete only, (b) abstract only, (c) concrete-to-abstract fading, or (d) abstract-to-concrete fading. Children in the concrete-to-abstract fading condition performed significantly better than children in the other conditions on transfer problems designed to assess understanding of mathematical equivalence. These results highlight the importance of fading from concrete to abstract representations. Specifically, children's understanding of mathematical concepts may benefit when problems are presented with concrete materials that are explicitly faded into more abstract symbolic representations.

Background

Working with abstract symbols to solve equations is a key factor for success in algebra. However, students often struggle to understand mathematical symbols. Consequently, some researchers propose introducing math concepts with concrete materials prior to presenting abstract symbols (Bruner, 1966; Burns, 1996; Tooke, et al., 1992).



For example, instead of writing "1 + _ = 3" on the board, a teacher may use a balance scale with one toy on the left side and three toys on the right. Various studies cite the potential benefits of using concrete materials including their ability to activate real world knowledge, prompt physical action, and facilitate connections between abstract concepts and real objects.

Background (cont.)

Conversely, others caution against the use of concrete materials because they often contain extraneous perceptual information, which can distract the learner from relevant structures, draw attention to irrelevant elements, and constrain the learners ability to apply the learned concept to novel problems (Goldstone & Sakamoto, 2003; Kaminski, et al., 2008)

However, many theorists who endorse using concrete materials specifically recommend beginning with concrete examples and slowly fading away to the more abstract (Bruner, 1966; Goldstone & Son, 2005).

We aim to test this "fading" method for teaching mathematical equivalence, an important concept in algebra. We hypothesize that our fading paradigm will foster greater understanding of the concept than purely concrete or abstract methods alone and will enhance transfer to novel equations.

Method

Participants
63 children who could not solve math equivalence problems correctly (ages 7-9; *M* age = 8 years, 5 months; 34 girls, 29 boys; 32% African American, 2% Asian, 21% Hispanic, 46% white)

Procedure
All children received instruction on the same six reflexive math equivalence problems (i.e., $a + b = a +$ __) in one of four conditions for 30 minutes during a one-on-one session with a tutor. The only difference between conditions was the context of the problem, which varied in terms of its concreteness relative to its abstractness. Children were assessed on their understanding of math equivalence immediately following instruction.

Conditions
Concrete Only – Children received instruction with concrete materials, first in the context of Monkey and Frog sharing stickers equally (3 problems) and then in the context of balancing objects on a scale (3 problems)

Abstract Only – Children received instruction with six abstract, symbolic math equivalence problems written on paper (e.g., 2 + 3 = 2 + __).

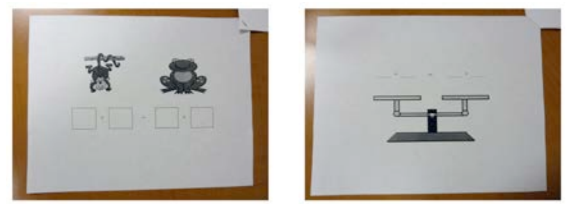
Method (cont.)

Concrete-to-Abstract Fading – Children received instruction in three contexts: first with concrete materials, then with "fading" worksheets, and finally with abstract problems. Three problems were taught in the context of Monkey and Frog sharing stickers equally and the last three problems followed this sequence in regard to balancing objects on a balancing scale.

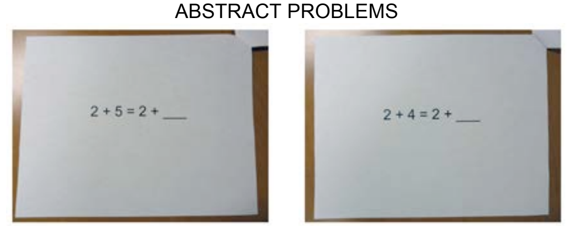
Abstract-to-Concrete Fading – Children received instruction in the same contexts as the concrete-to-abstract fading condition, but in the reverse order. Thus, they were first taught with abstract problems, then with the "fading" worksheet, and finally with concrete materials.

CONCRETE MATERIALS

FADING WORKSHEETS



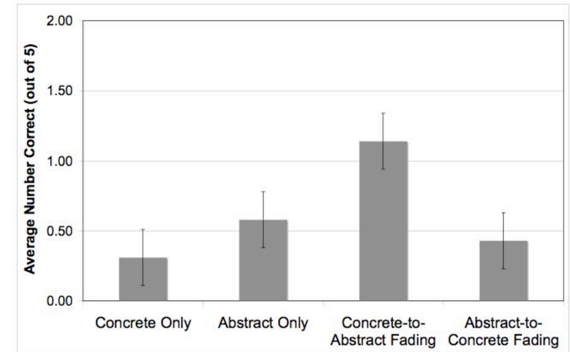
ABSTRACT PROBLEMS



Transfer Test
Five transfer problems designed to assess understanding of mathematical equivalence. Children heard: "These problems are just like the problems we just solved together. You should think about them the same way."

Results

Performance on Transfer Test By Condition



- There was a main effect of instruction condition, $F(3, 63) = 2.80$, $p = .048$.
- As predicted, children in the concrete-to-abstract fading condition performed significantly better on transfer problems designed to asses understanding of mathematical equivalence than children in the other three conditions, $F(1, 63) = 7.68$, $p = .007$.

Summary and Implications

- Children constructed a better understanding of mathematical equivalence after receiving instruction with concrete materials that faded away to more abstract representations compared to either purely abstract or concrete methods alone.
- The "fading" method may be helpful because it takes advantage of the benefits of concrete materials but still explicitly fades to abstract materials so as to promote transferable knowledge that can be applied across contexts and domains.
- The "fading" method may be used effectively during classroom instruction as a way to improve students' understanding of mathematical symbols and thus enhance their capacity for success in algebra.